Quadcopter Trajectory Tracking Control using State-Feedback Control with Integral Action

Ahmed Samir
Faculty of Engineering at Shoubra, Benha University, Egypt
108 Shoubra, Cairo Governorate, Egypt

Abdallah Hammad
Faculty of Engineering at Shoubra, Benha University, Egypt
108 Shoubra, Cairo Governorate, Egypt

Ashraf Hafez
Faculty of Engineering at Shoubra, Benha University, Egypt
108 Shoubra, Cairo Governorate, Egypt

Hala Mansour
Faculty of Engineering at Shoubra, Benha University, Egypt
108 Shoubra, Cairo Governorate, Egypt

ABSTRACT

Trajectory tracking of Unmanned Aerial Vehicles (UAVs) is a very challenging and complicated field of research due to their nonlinear and underactuacted dynamics. In this paper, a real time trajectory tracking controller is developed for a quadrotor. A state feedback with integral action control scheme is designed for the position controller to ensure that the quadrotor can track the reference position rapidly. The quadrotor dynamic model is established using Newton-Euler formalism taking into account various physical phenomena that can effect its dynamic behavior. NI-LabVIEW based simulation results show that the proposed controller can make the quadrotor tracks the desired trajectory quickly and smoothly with ensuring the stability for roll and pitch angle.

Keywords

Quadcopter, Realtime, Tracking, Navigation, Trajectory, Modling, State feedback

1. INTRODUCTION

More recently, quadrotor UAV becomes very popular. Taken broadly, it is a sort of new kind of unmanned aerial vehicle. But compared to the traditional UAV, quadrotor is simpler, smaller ,and has lower cost. It can not only fly in three-dimensional space flexibly, but also can gather information through on-board sensors. Due to that, it has been broadly utilized in several field, such as rescue missions, aerial photography, investigation ,and mapping especially in narrow space and more risky environment [1]-[3].
Quadrotor has six degrees of freedom in space, but because of the restriction of structure, it can only be using four independent motors, so the quadrotor is a sort of typical under-actuated system.
The flight mechanism is as the following: vertical movement is made by collectively increasing and decreasing the equally speed of every rotor. Pitch movement is accomplished by the differential speed of the front back set of rotors. Roll is accomplished by the differential speed of the left right set of rotors. Yaw motion is realized by the different reactive torques [4].
As specified, many common control algorithms for UAV are not applicable for quadrotor due to its under-actuation. So late years, control methodology and engineering application for quadrotor has gotten more consideration of scholars. These kinds of UAVs have a high nonlinear and time-varying behavior, parametric uncertainties and are affected by atmospheric turbulence. Therefore, a robust adaptive controller is required to achieve accurate tracking and high performances in an autonomous flight with disturbance rejection capabilities [5].
The problem of trajectory tracking has been proposed of several controller. Many traditional control techniques have been applied such as PD control[6]. PID control [7][8] and LQR control method [8].Nevertheless, PD and PID control algorithm were used to understand the tracking control of quadrotor in [6][7]. To reduce the steady state error, the gain of PID must be increased but the system was easily affected by outer disturbance.
The underactuated trajectory tracking problem was handled using backstepping in [9] and sliding mode control techniques in [10][11]. In [12] the authors gave a solution to the time scale separation assumption by means of a tracking differentiator and used a new command filtered compensation for quadrotor trajectory control. Nested saturation algorithm combined with backstepping and feedback control technique has been used in [13].
The authors in [14] proposed a nonlinear adaptive tracking controller based on backstepping techniques that can estimate and compensate the effect of mass uncertainty. In [15] a state feedback control system and an integrator backstepping approach was used for tracking control. This paper presents implementation of a real time trajectory tracking controller using state feedback control with integral part.
This paper is organized as follows: The current section is introduction. Section 2 presents a description of the basic structure, and dynamic model of quadrotor. The state feedback controller with integral part architecture is introduced in section 3. Section 4 presents simulation results to validate the proposed controller. The conclusions and the future work are given in section 5.

2. DYNAMIC MODEL OF QUADROTOR

The quadcopter depicted in Fig. 1 consists of a six degree of-freedom(DOF) body actuated by four propellers. The four rotors provide upwards thrusting as well direction control. The motors are located at the four corners of the quadcopter. The motors situated diagonally opposite to each other rotate in the same direction while the adjacent motors rotate in the counter direction. The mathematical model was developed using Newton-Euler dynamic equations using the following simplifying assumptions [16] :

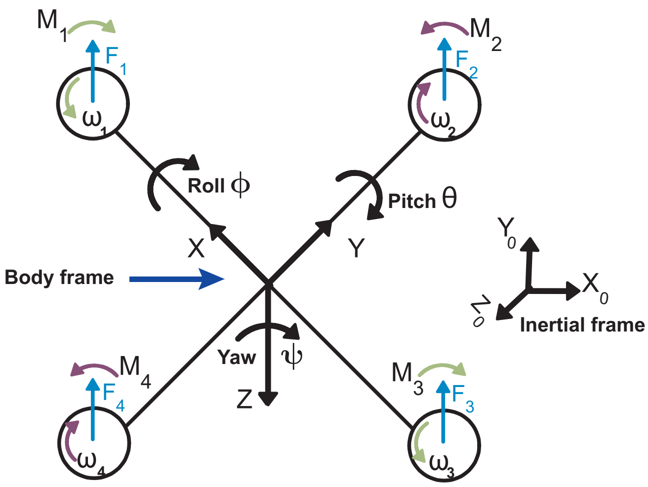

Fig. 1. A typical structure of quadrotor

—The structure including the motors are rigid.

—The gyroscopic effects of the propellers are negligible.

—The quadcopter has two planes of symmetry, i.e., the products of inertia are equal to zero, $Ixy = Iyz = Ixz$.

The mathematical model are given as follows :

$$\ddot{\phi} = \frac{L}{I_{xx}}U_2 - \frac{J_r}{I_{xx}}\omega_r\dot{\theta} + \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\psi}\dot{\theta} \quad , \tag{1}$$

$$\ddot{\theta} = \frac{L}{I_{yy}}U_3 - \frac{J_r}{I_{yy}}\omega_r\dot{\phi} + \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\psi}\dot{\phi} \quad , \tag{2}$$

$$\ddot{\psi} = \frac{1}{I_{zz}}U_4 + \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\phi}\dot{\theta} \quad , \tag{3}$$

$$\ddot{x} = \frac{-U_1}{m}(\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta) \quad , \tag{4}$$

$$\ddot{y} = \frac{-U_1}{m}(\cos\phi\sin\psi\sin\theta - \sin\phi\cos\psi) \quad , \tag{5}$$

$$\ddot{z} = g - \frac{U_1}{m}(\cos\phi\cos\theta) \quad . \tag{6}$$

where the parameters using in eq. (1) - (6) are described in table (1).
The system has six outputs(x,y,z,ϕ,θ,ψ)and four inputs(U_1, U_2, U_3, U_4). The control input U_1 provides thrust on the body in the z-axis, U_2 and U_3 are the roll and pitch inputs and U_4 is used for yaw control. The control inputs U_i can be written in terms of rotor speeds ω_r as:

Table 1. **Variable description**

Parameter	Unit	Descritpion
ϕ	rad	Roll angle
θ	rad	Pitch angle
ψ	rad	Yaw angle
ω_r	rad/sec	Angular velocity of the propeller blades
J_r	$kg.m^2$	Moment of inertia of the propeller blades
I_{xx},I_{yy},I_{zz}	$kg.m^2$	Moments of inertia of the quadcopter
L	m	Length from the rotors to the center of mass
m	kg	Mass of the quadrotor
U_i	N	Control inputs
K_F	N.s^2	Aerodynamic force coefficient
K_M	N.m.s^2	Aerodynamic moment coefficient

$$U = \begin{cases} U_1 = -k_f(\omega_1^2+\omega_2^2+\omega_3^2+\omega_4^2) \\ U2 = k_f L(-\omega_2^2+\omega_4^2) \\ U3 = k_f L(\omega_1^2-\omega_3^2) \\ U_4 = k_M L(\omega_1^2-\omega_2^2+\omega_3^2-\omega_4^2) \end{cases} \tag{7}$$

To transform the coordinates from body-fixed frame of quadcopter to Earth fixed frame the rotation matrix R is used and is given below:

$$R = \begin{pmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{pmatrix} \tag{8}$$

where C and S denote cos and sin respectively.

3. QUADCOPTER CONTROLLER DESIGN

State feedback control [21] is a strategy utilized in feedback control system theory to put the closed-loop poles of a plant in pre-decided areas in the s-plane. Placing poles is desirable in light of the fact that the area of the poles relates straightforwardly to the eigenvalues of the system, which control the qualities of the reaction of the system. This method is broadly utilized as a part of system with multiple inputs and multiple outputs. Transfer function can be represented by the state space equation:

$$\dot{x} = Ax + BU \quad , \tag{9}$$

with output equation :

$$y = Cx + DU \quad , \tag{10}$$

then the poles of the system are the roots of the characteristic equation given by :

$$|sI - A| = 0 \quad , \tag{11}$$

state feedback is used by dominant the input vector u :

$$U = -K_0 x \quad , \tag{12}$$

substituting into eq. (9) and (10) :

$$\begin{aligned} \dot{x} &= (A - BK_0)x \\ y &= (C - DK_0)x \end{aligned} \quad , \tag{13}$$

the feedback matrix K_0 which drive the closed-loop eigenvalues to the pole position fixed by the required characteristic equation.

3.1 State Feedback with Integral Control

Originally, integral action was utilized in controller design to minimize steady state errors. Much of the time, due to in precision of linearization of quad model, the controller gains cann't be adjusted precise and there will be a steady state error. The solution is to guarantee an integral part in the controller design. Fig. 2 shows controller block diagram.

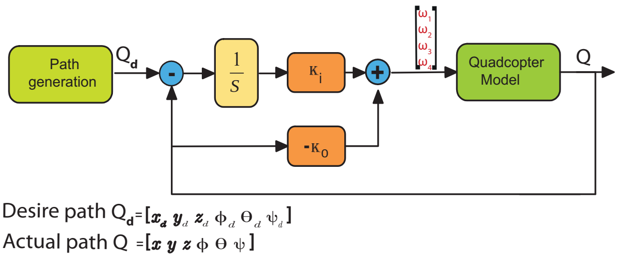

Fig. 2. Controller block diagram

4. SIMULATION RESULTS

The simulation results are obtained by using NI-LabVIEW. Simulations were carried out using several scenarios in order to verify the performance of the proposed controller for the trajectory tracking problem. The values of the model parameters used in the simulations are shown in Table 2. The controller parameters are chosen as : integral gain matrix,

$$K_i = \begin{bmatrix} 0 & 0 & 0 & 0.4 & 0 & 0.35 \\ 0 & 0 & 0 & 0.4 & 0 & -0.35 \\ 0 & 0 & 0 & 0.4 & 0.35 & 0 \\ 0 & 0 & 0 & 0.4 & -0.35 & 0 \end{bmatrix} \tag{14}$$

and state feedback matrix,

$$K_0 = \begin{bmatrix} -0.4 & -10^{-4} & 67.7 & 3.9 & -10^{-5} & 1.4 \\ -0.4 & -10^{-4} & -67.7 & 3.9 & -10^{-5} & -1.4 \\ 0.4 & -67.6 & -10^{-4} & 3.9 & 1.4 & 10^{-5} \\ 0.4 & 67.6 & -10^{-4} & 3.9 & -1.4 & -10^{-5} \end{bmatrix} \tag{15}$$

The sampling rate is 100 Hz.

Table 2. **Quadrotor model parameters**

Symbol	Definition	value	Unit
m	Mass	1.3	Kg
g	Local gravity	9.8	m/s^2
k_f	Aerodynamic force Constant	1.08E-06	$N.s^2$
k_m	Aerodynamic moment Constant	1.08E-08	$N.m.s^2$
I_{xx}	Moment of inertia about x	0.128	$kg.m^2$
I_{yy}	Moment of inertia about y	0.128	$kg.m^2$
I_{zz}	Moment of inertia about z	0.256	$kg.m^2$
L	Moment arm	0.21	m

4.1 Way-Points Tracking :

The position of the next way-point is sent to position controller which directs the UAV towards the goal. Fig. 3,4 show a square trajectory defined by four way-points that described in table 3. The task was climbing to 2m from the ground and follow the four way-points of a square of 5m side. In order to track the square trajectory, the controller takes the (x_d, y_d) position references, and consequently the position controller produces the (θ_d,ϕ_d) attitude references. Fig. 3.(a),(b) show the tracking response in y and x directions. Fig. 3.(c),(d) show the attitude response. Fig. 4.(a) shows the altitude response. Fig. 4.(b) shows the yaw response. Fig. 4.(c),(d) show the tracking performance of the quad in 2d-plan and 3d-plan.

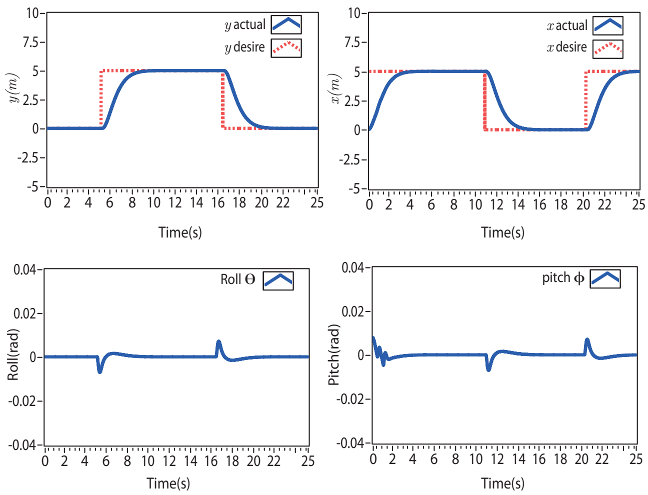

Fig. 3. (a) The desired and the actual values of y. (b) The desired and the actual values of x. (c) the roll response. (d) the pitch response.

Table 3. **Way points**

start		(0,0,0)
Point1	P1	(5,0,2)
Point2	P2	(5,5,2)
Point3	P3	(0,5,2)
Point4	P4	(0,0,2)

Table 4 shows a summary of the performance of the system in terms of its settling time and overshoot.

Table 4. **Controller Response Results**

	Desired Value	Settling Time	Overshoot
Altitude (z)	2 m	1.4 sec	1.40%
Attitude (θ)and(ϕ)	2	0.5 sec	2%
Position (x)and(y)	5 m	4 sec	0%

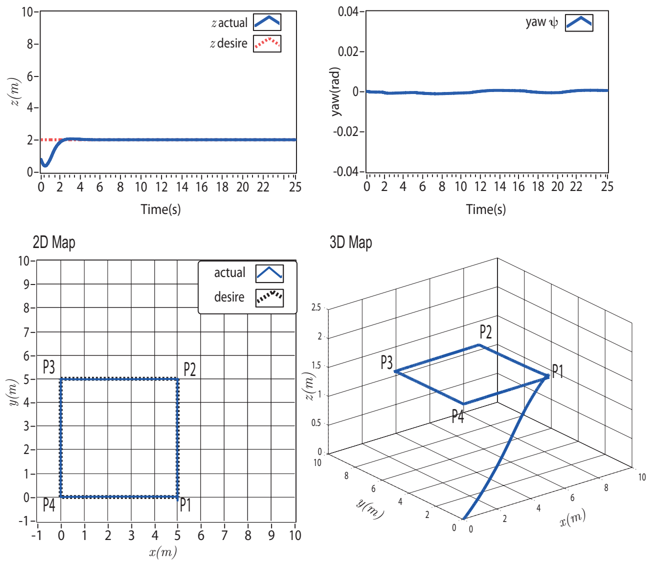

Fig. 4. Simulation result for tracking multi-point. (a) the altitude response. (b) the yaw response. (c) The tracking performance in 2d. (d) The tracking performance in 3d.

4.2 Line Path Tracking :

Fig. 5,7 show a line trajectory tracking defined by two points (start point and end point). The task was climbing to 2m from the ground (0,0,0) and follow the line between point(0,0,2)m to point (5,5,2)m. Fig. 5.(a),(b) show the tracking response in y and x directions. Fig. 6.(a),(b) show the attitude response. Fig. 7.(a) shows the altitude response. Fig. 7.(b) shows the yaw response. Fig. 7.(c),(d) show the line tracking performance of the quad in 2d-plan and 3d-plan.

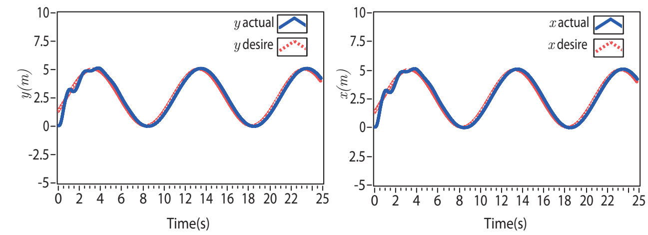

Fig. 5. (a) The desired and the actual values of y. (b) The desired and the actual values of x.

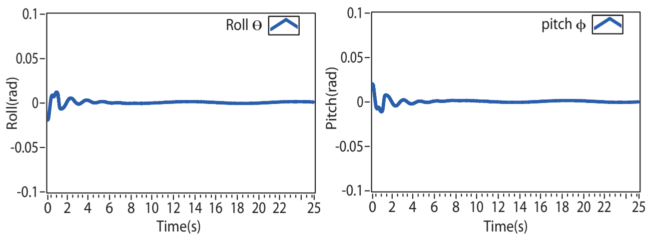

Fig. 6. (a) the roll response. (b) the pitch response.

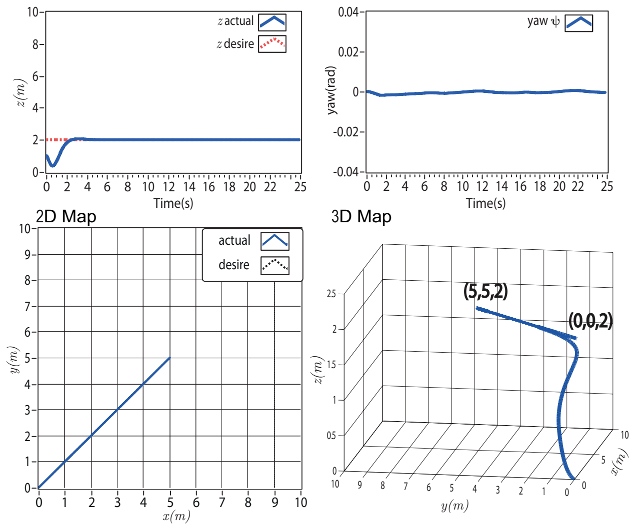

Fig. 7. Simulation result of line tracking. (a) the altitude response. (b) the yaw response. (c) The tracking performance in 2d. (d) The tracking performance in 3d.

4.3 Circular Path Tracking :

The task was climbing to 2m from the ground and follow the a circle of center (3,3)m and radius (3m). the path described as $x_d(t) = 3 + 3\sin 0.2\pi t$ and $y_d(t) = 3 - 3\cos 0.2\pi t$, or in general $(x_d - 3)^2 + (y_d - 3)^2 = 9$. Fig. 8.(a),(b) show the tracking response in y and x directions. Fig. 9.(a),(b) show the attitude response. Fig. 10.(a) shows the altitude response. Fig. 10.(b) shows the yaw response. Fig. 11.(a),(b) show the circular tracking performance of the quad in 2d-plan and 3d-plan.

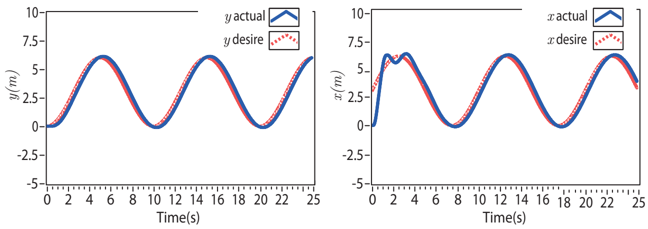

Fig. 8. The position signals generated to track the circle trajectory. (a) The desired and the actual values of y. (b) The desired and the actual values of x.

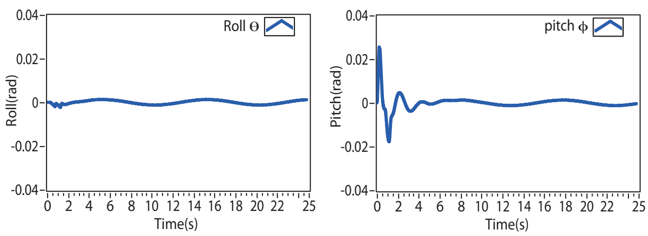

Fig. 9. The attitude signals generated to track the circle trajectory (a) the roll response. (b) the pitch response.

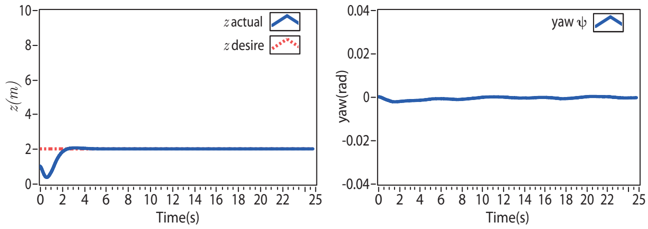

Fig. 10. (a) the altitude response. (b) the yaw response.

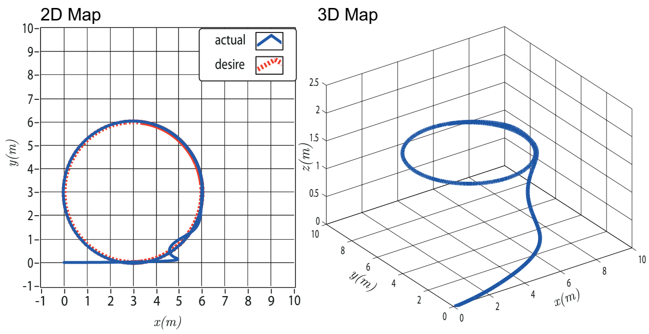

Fig. 11. Simulation result of circular path tracking. (a) The tracking performance in 2d. (b) The tracking performance in 3d.

4.4 Ellipsoidal Path Tracking :

The task was climbing to 2m from the ground and follow the a ellipse. The path described as $x_d(t) = 3 + 3\sin 0.2\pi t$ and $y_d(t) = 3 - 3\cos(0.2\pi t + \pi/4)$. Fig. 12.(a),(b) show the tracking response in y and x directions. Fig. 13.(a),(b) show the attitude response. Fig. 14.(a) shows the altitude response. Fig. 14.(b) shows the yaw response. Fig. 15.(a),(b) show the ellipse tracking performance of the quad in 2d-plan and 3d-plan.

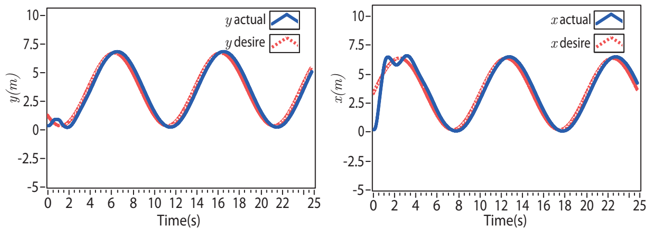

Fig. 12. The position signals generated to track the ellipse trajectory. (a) The desired and the actual values of y. (b) The desired and the actual values of x.

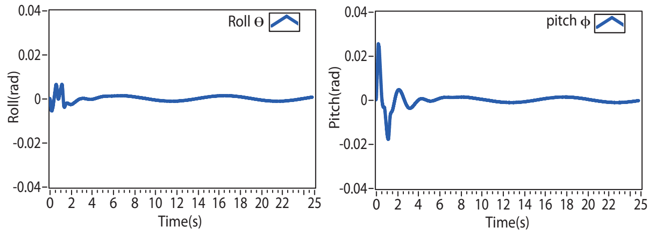

Fig. 13. The attitude signals generated to track the ellipse trajectory. (a) the roll response. (b) the pitch response.

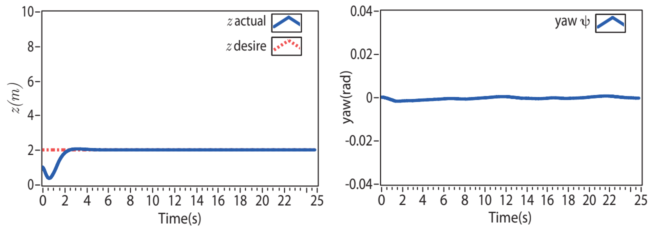

Fig. 14. (a) the altitude response. (b) the yaw response.

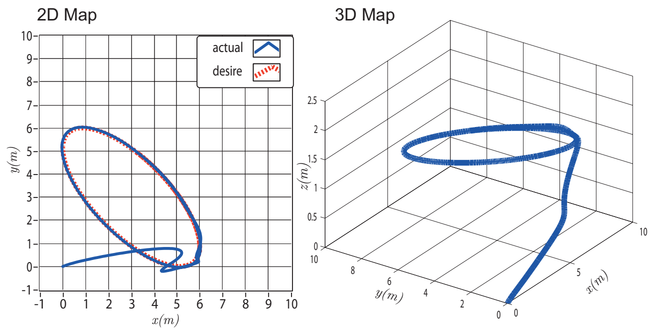

Fig. 15. Simulation result of ellipse path tracking. (a) The tracking performance in 2d. (b) The tracking performance in 3d.

4.5 Figure-Eight Path Tracking :

The task was climbing to 2m from the ground and then follow the figure eight Path. The path described as :$x_d(t) = 3 + 3\sin(0.2\pi t)$ and $y_d(t) = 3 + 3\cos(0.1\pi t)$. Fig. 16.(a),(b) show the tracking response in y and x directions. Fig. 17.(a),(b) show the attitude response. Fig. 18.(a) shows the altitude response. Fig. 18.(b) shows the yaw response. Fig. 19.(a),(b) show the figure eight tracking performance of the quad in 2d-plan and 3d-plan.

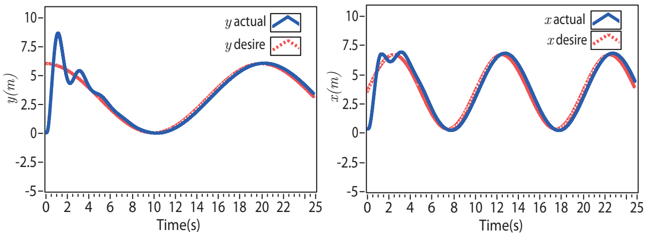

Fig. 16. The position signals generated to track the figure eight trajectory. (a) The desired and the actual values of y. (b) The desired and the actual values of x.

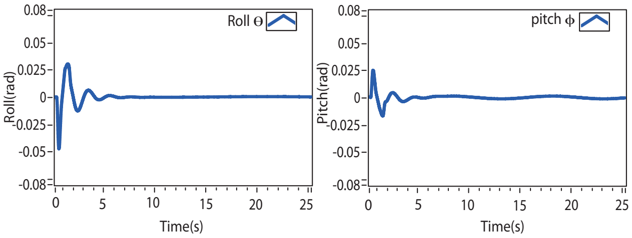

Fig. 17. The attitude signals generated to track the figure eight trajectory. (a) the roll response. (b) the pitch response.

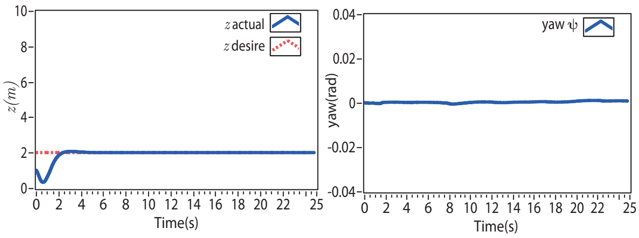

Fig. 18. (a) the altitude response. (b) the yaw response.

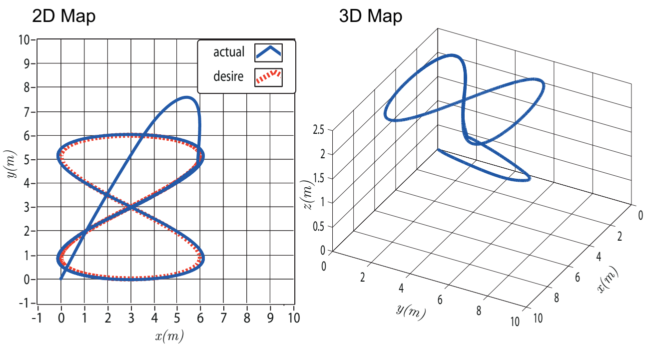

Fig. 19. Simulation result of figure eight path tracking. (a) The tracking performance in 2d. (b) The tracking performance in 3d.

5. CONCLUSIONS

In this paper a real time trajectory tracking based on state feedback with integral action control scheme is developed for a quadrotor. In order to test the performance of the designed controller, the kinematic and dynamic models of quadrotor are included to implement simulation in NI-LabVIEW. The simulation results for several scenarios show that all the state variables converge to their reference values. To validate the proposed controller practically, several experiments are under development in our lab.

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